LIQUID TEMPERATURE DEPENDENT BEHAVIOUR OF A CAVITATION BUBBLE IN ACOUSTIC FIELD

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keywords: Acoustic bubble, Cavitation, Ultrasonic

Abstract

A new model of bubble dynamics is constructed using linear wave equation, including effects of variation of the gas temperature inside the bubble and the liquid temperature near the bubble, and effects of evaporation-condensation of the liquid vapour at the bubble wall. The liquid is assumed water and the gas inside the bubble is only vapour (neglecting non-condensable gas). The temperature inside the bubble and the liquid temperature are numerically calculated by solving the energy equation both inside (vapour-phase) and outside (liquid-phase) the bubble (using finite difference method). The pressure inside the bubble is obtained numerically without assuming that it follows any assuming relation. The results reveal that the bubble radius, the liquid temperature, and the pressure and temperature inside the bubble change with time periodically. Both the pressure and temperature become higher when the radius becomes minimum. The present theoretical result is compared with data from other reference and with another theoretical model to check the validity of the present model. The calculated result approximately fits with the data of the previous studies.

التصرف المعتمد لدرجة حرارة السائل لفقاعة التفقع في وسط صوتي

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الخلاصة

تم إنشاء نموذج جديد لحركة الفقاعة باستخدام المعادلة الموجية الخطية، متضمنا تأثيرات تغير درجة حرارة الغاز داخل الفقاعة ودرجة حرارة السائل القريبة من الفقاعة وتأثيرات تبخير وتكثيف بخار السائل عند جدار الفقاعة. أُفترض السائل هو الماء والغاز داخل الفقاعة هو البخار فقط (إهمال الغاز غير الذائب). تم حساب عدديا درجة الحرارة داخل الفقاعة ودرجة حرارة السائل بواسطة حل معادلة الطاقة داخل الفقاعة (طور البخار) وخارج الفقاعة (طور السائل) (باستخدام طريقة الفروقات المحددة). حسب الضغط داخل الفقاعة عدديا بدون أن نفترض انه يتبع أي علاقة مفترضة. تبوح النتائج أن نصف قطر الفقاعة ودرجة حرارة السائل والضغط ودرجة الحرارة داخل الفقاعة يتغيرون مع الزمن بشكل دوري. يصبح كل من الضغط ودرجة الحرارة كبيرا عندما يصبح نصف القطر صغيرا. تم مقارنة النتائج النظرية المقدمة مع بيانات من مصدر أخر ونموذج رياضي أخر للتحقيق من صحة النموذج المقدم. تطابق تقريبي للنتائج المحسوبة مع البيانات للدراسات السابقة.

1. Introduction

The dynamical behavior of bubbles in variable pressure fields in liquids is of central interest in variety of situations of technological and scientific relevance such as the propagation of pressure pluses in gas-liquid or vapour-liquid mixtures, acoustic cavitation, ultrasonic cleaning, vibration-augmented heat transfer, and others [1].

Rayleigh made the first analysis of a problem in cavitation and bubble dynamics in 1917. He solved the problem of the collapse of an empty cavity in a large mass of liquid. Rayleigh also considered in this paper the problem of a gas-filled cavity under the assumption that the gas undergoes an isothermal compression. His interest in these problems presumably arose from concern with cavitation and cavitation damage. Neglecting the surface tension and liquid viscosity and with the assumption of liquid incompressibility, Rayleigh showed from the momentum equation that the bubble boundary R (t) obeyed the relation [2].

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{P_{L,R} - P_{L,\infty}}{\rho_L} \qquad \ldots (1)$$

A gas bubble in a liquid performs forced radial oscillations when a sound wave impinges upon it. Large amplitudes result when the acoustic frequency is at or near the bubbles natural frequency, or certain rational multiples of it. Since these large oscillations can be important in cavitation. Oscillations of large bubbles were originally analyzed by Plesset as is reviewed in ref.[3], Noltingk and Neppiras [4], and Poritsky [5] modified this equation (eq. (1)) to include the effects of viscosity, surface tension, and an incident sound wave, and it was this modified equation which Lauterborn [6] solved. A different modification was made by Keller and Kolodner [7], they included the effects of acoustic radiation by treating the surrounding liquid as slightly compressible.

A new equation for the bubble radius will be derived by depending on some modifications. It includes the effects of acoustic radiation, viscosity, surface tension, compressibility, thermal conduction inside the bubble, variation of liquid temperature near bubble wall, and mass transfer (evaporation and condensation) at bubble wall. In this paper, a new model of bubble dynamics is constructed including effect of variation of liquid temperature at bubble wall, that of non-equilibrium evaporation and condensation of water vapour at bubble wall, and that of thermal conduction inside a bubble.

It should be noted that the liquid temperature near bubble wall has already been calculated in some theoretical studies of bubble dynamics in acoustic field [8]. However, in the studies, the latent heat of intense vapour condensation is completely neglected. Researchers in ref. [9] have already constructed models of bubble dynamics including effect of the latent heat of evaporation and condensation. However, the new model constructed in this paper differs from the previous model [9] in many points. In Ref. [9], it is assumed that the vapour pressure inside a bubble (P_V) is at the equilibrium vapour pressure (P_V^*) at the temperature of the bubble wall, while in the present study non-equilibrium effect of evaporation and condensation is taken into account by solving the heat conduction equation numerically without assuming a profile of liquid temperature.

2. Mathematical Formulation

Bubble size at a stable condition in acoustic field depends on several experimental parameters such as the acoustic pressure amplitude and the liquid temperature. Bubble's size is determined by mass exchange of vapour between the interior of the bubble and its surrounding. Rate of mass exchange depends on the liquid temperature at bubble wall [10]. In this section, effect of variation of liquid temperature at bubble wall is included in the model of bubble dynamics in acoustic field.

From the point of view of classical fluid mechanics, a bubble in a liquid is a free-boundary problem in which the mechanical and thermal behavior of the fluids -liquid and vapour- is described by the usual conservation equations coupled by suitable conditions at the gas-liquid interface. The position of this interface is it self-unknown a priori and is determined in the process of applying the interface conditions. In its generality the problem is therefore complex and only amenable to numerical calculations [11].

We shall assume that a spherical vapour bubble of the initial radius R_o with uniform interior is placed at a fixed point in liquid. Moreover, the following assumptions shall be applied as follows:

(1) The pressure inside the bubble is uniform.
(2) The bubble contains only vapour.
(3) The bubble is spherically symmetric.
(4) The vapour obeys the perfect-gas law.
(5) The effect of gravity and diffusion of air is neglected.
(6) The liquid is compressible.

(7) The temperature inside and outside the bubble is not uniform.

(8) The physical properties of the vapour and liquid are variable.

Let $\phi(r,t)$ be the velocity potential for the liquid. The linear wave equation is [12]

$$\frac{\partial^2 \phi}{\partial r^2}+\frac{2}{r}\frac{\partial \phi}{\partial r}-\frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2}=0 \qquad \ldots(2)$$

Equation of bubble radius is derived here including effects of evaporation and condensation (mass transfer) and variation of liquid temperature at bubble wall. The derivation depends on Keller and Kolodner approach [7] with the following two boundary conditions [13,14]

$$\left(\frac{\partial\phi}{\partial r}\right)_{r=R}=\dot{R}+\frac{\dot{m}}{\rho_{L,R}} \qquad \ldots(3)$$

$$\left[\frac{\partial \phi}{\partial t}+\frac{1}{2}\left(\frac{\partial \phi}{\partial r}\right)^2\right]_{r=R}=-H \qquad \ldots(4)$$

where r is the radius from the center of the bubble, $\dot{R}$ is the velocity of the bubble wall, and the other symbols have appeared in nomenclature. The general solution of eq. (2) is given by eq. (5) under the spherical symmetry.

$$\phi(r,t)=\frac{1}{r}F(t-\frac{r}{c}) \qquad \ldots(5)$$

Let

$$\left.\begin{aligned} y&=t-\frac{r}{c}\\ F'&=\frac{dF}{dy}\\ \bar{y}&=t-\frac{R}{c}\end{aligned}\right\} \qquad \ldots(6)$$

From eqs. (3) and (5):

$$-\frac{1}{Rc}F'(\bar{y})-\frac{1}{R^2}F(\bar{y})=\dot{R}-\frac{\dot{m}}{\rho_{L,R}} \qquad \ldots(7)$$

and from eqs. (4) and (5):

$$\frac{1}{R}F'(\bar{y})+\frac{1}{2}\left(\dot{R}-\frac{\dot{m}}{\rho_{L,R}}\right)^2=-H \qquad \ldots(8)$$

From eq. (8)

$$\frac{F'(\bar{y})}{R}=-H-\frac{1}{2}\left(\dot{R}-\frac{\dot{m}}{\rho_{L,R}}\right)^2 \qquad \ldots(9)$$

and from eq. (7)

$$\frac{F(\bar{y})}{R^2}=\frac{1}{2c}\left(\dot{R}-\frac{\dot{m}}{\rho_{L,R}}\right)^2-\dot{R}+\frac{\dot{m}}{\rho_{L,R}}+\frac{H}{c} \qquad \ldots(10)$$

Differentiating eq. (10) w.r.t. t and multiplying by R yields eq. (11).

$$R\ddot{R}\left(1-\frac{\dot{R}}{c}+\frac{\dot{m}}{c\,\rho_{L,R}}\right)-\frac{R\ddot{m}}{\rho_{L,R}}\left(\begin{array}{c}1-\frac{\dot{R}}{3c}+\\ \frac{\dot{m}}{c\,\rho_{L,R}}\end{array}\right)$$

$$+\frac{3}{2}\dot{R}^2\left(1-\frac{\dot{R}}{3c}+\frac{2\dot{m}}{3c\,\rho_{L,R}}\right)-\frac{\dot{m}}{\rho_{L,R}}$$

$$\left[\begin{array}{l}\dot{R}+\frac{\dot{m}}{2\rho_{L,R}}+\frac{\dot{m}\dot{R}}{2c\,\rho_{L,R}}-\frac{d\rho_{L,R}}{dt}\\ \left(\frac{R}{\rho_{L,R}}+\frac{\dot{m}R}{c\,\rho^2_{L,R}}-\frac{R\dot{R}}{c\,\rho_{L,R}}\right)\end{array}\right]$$

$$=\left(1+\frac{\dot{R}}{c}\right)H+\frac{R}{c}\frac{dH}{dt} \qquad ...(11)$$

where H is the liquid specific enthalpy difference between the bubble wall and the infinity. It is given by [13]:

$$H=\frac{1}{\rho_{L,\infty}}(P_{L,R}-P_{L,\infty}) \qquad ...(12)$$

$P_{L,R}$ is related to the pressure inside the bubble (P_v (t)) by eq. (13) [15]

$$P_{L,R}=P_V(t)-\frac{2\sigma}{R}-\frac{4\mu}{R}\left(\dot{R}-\frac{\dot{m}}{\rho_{L,R}}\right)-\dot{m}^2\left(\frac{1}{\rho_{L,R}}-\frac{1}{\rho_{v,R}}\right) \qquad ...(13)$$

The net rate of evaporation per unit area and unit time ($\dot{m}$) is given by [16]

$$\dot{m}=\beta\frac{P_v^*(T_{L,R})-P_v(t)}{\sqrt{2\pi\,R_v\,T_{L,R}}} \qquad ...(14)$$

When a bubble is irradiated by an acoustic wave of which wave length is much larger than the bubble radius, $P_{L,\infty}$ is given by [17]

$$P_{L,R}=P_\infty-P_m\,\sin(\omega t) \qquad ...(15)$$

Following are the description of calculated $P_v(t)$. The mathematical model for the bubble interior is described in detail in Prosperetti *et al.* [18], Kamath and Prosperetti [19], and Prosperetti [20]. The model accounts for the compressibility of the vapour and heat transport by convection and by conduction inside the bubble.

The internal pressure P_v is found by integrating [21].

$$\frac{dP_v}{dt}=\frac{3}{R}\left[(\gamma-1)\left(k_v\frac{\partial T_v}{\partial r}\right)_{r=R}-\gamma P_v\dot{R}\right] \qquad ...(16)$$

where γ is the ratio of the specific heats of the vapour and k_v is the vapour thermal conductivity. The vapour temperature field $T_v(r,t)$ is obtained from:

$$\frac{\gamma}{\gamma-1}\frac{P_v}{T_v}\left[\frac{\partial T_v}{\partial t}+u_v\frac{\partial T_v}{\partial r}\right]=\frac{dP_v}{dt}+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,k_v\frac{\partial T_v}{dr}\right) \qquad ...(17)$$

with

$$u_v = \frac{1}{\gamma P_v}\left[(\gamma-1)\,k_v\frac{\partial T_v}{\partial r} - \frac{1}{3}r\frac{dP_v}{dt}\right] \quad \dots (18)$$

Finally, since the effect of liquid compressibility in the convective term of the energy balance in the liquid is neglected [13].

$$\rho_L c_{PL}\left[\frac{\partial T_L}{\partial t} + \frac{R^2}{r^2}\dot{R}\frac{\partial T_L}{\partial r}\right] = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k_L \frac{\partial T_L}{\partial r}\right) \quad \dots (19)$$

The model is completed by energy equation in each phase (eqs. (17), (19) and by the equation for bubble motion (eq.(11)). This model is then solved using a finite difference technique with the boundary conditions at the interface (r =R)

$$T_v = T_L \quad \dots (20)$$

$$k_L\frac{\partial T_L}{\partial r} = k_v\frac{\partial T_v}{\partial r} \quad \dots (21)$$

and

$$\text{at } r = 0 \quad \frac{\partial T_v}{\partial r} = 0 \quad \dots (22)$$

$$\text{at } r \to \infty \quad T_L = T_O \quad \dots (23)$$

Physical quantities of liquid depend on the liquid temperature (T_L) and the liquid pressure (P_L). Formulas of the quantities employed in the calculations are described in Appendix in ref. [14].

3. Results and Discussion

Calculations are performed under the following conditions. The ambient liquid temperature (T_o) and the ambient liquid pressure (P_o) are chosen to be 20 °C and 1 bar, respectively. At these conditions, the speed of sound in liquid water (c) and the specific heat of liquid water at constant pressure (C_{PL}) are 1983 (m/s) and 4.2 (kJ/kg. K), respectively. The accommodation coefficient for evaporation and condensation (β) is assumed to be β= 0.4 [9]. The initial bubble radius R_o=8 μm and the acoustic pressure amplitude P_m=1.25 bar [22].

Results of the calculation are shown in Figs. (1 ~ 6) for two acoustic cycles. All the physical quantities of a bubble change periodically with the frequency of the acoustic field applied on the bubble.

In Fig. 1, the bubble radius (R) increases with decreasing the ambient liquid pressure. It decreases when the ambient liquid pressure is increasing. The ambient liquid pressure increases and decreases due to change the pressure of the sound field. When the radius decreases, the liquid temperature at the bubble wall ($T_{L,R}$) will be increasing as shown in Fig. 2. This is occurred due to increase the vapour pressure (P_v) inside the bubble as shown in Fig. 3. When the vapour pressure increases, intense vapour condensation takes place at the bubble wall. Therefore, the liquid temperature at the bubble wall is increasing. Also, it is caused by the thermal conduction from the heated interior of the bubble to the surrounding liquid. This is occurred due to increase the temperature inside the bubble (T_v) as shown in Fig. 4. From Figs. 2-4, at collapse of the bubble, the liquid temperature, and the pressure and temperature inside the bubble increase suddenly because the radius becomes minimum. They are followed by small oscillations due to the small bounces of the radius (see Fig. 1). High temperature inside the bubble is occurred at the center of the bubble and it equals to the liquid temperature at the bubble wall as shown in Fig. 5. This is occurred due to the thermal conduction inside the bubble.

In Fig. 6, the radius-time curve which calculated from the present model is compared with the experimental data from reference [22]. In this reference, the quantities or conditions are the same values which is used in the present calculation. Also, in this figure, direct comparison is given between the present calculation and the theoretical result of Lastman-Wentzell model [23] which is derived from nonlinear wave equation (also, see Figs. 7 and 8). The present calculated result approximately fits with the experimental data. These comparisons give the validity of the present model which is derived from linear wave equation.

4. Conclusion

The results obtained in the present paper may be summarized as follows. A new model of bubble dynamics in acoustic field is derived from linear wave equation. It is including effects of variation of the vapour temperature inside the bubble and the liquid temperature at the bubble wall, and the effects of evaporation-condensation of the vapour inside the bubble. The temperature inside and outside the bubble are changing due to change the bubble radius. High temperature occurs at the bubble center and it decreases at the bubble wall. This is occurred due to the thermal conduction inside the bubble. The liquid temperature at the bubble wall increases when the radius becomes minimum. This is occurred because the thermal conduction and the condensation at the bubble wall.

The calculated result of the present model approximately fits with the data of the previous studies. Therefore, the present model in this study can be used to study oscillations of a bubble in any liquid.

5. Nomenclature

Subscripts

v: Refers to the bubble content (water vapor)

L: Refers to liquid

L,R: Refers to liquid at bubble wall

o: Refers to the equilibrium state

∞: Refers to conditions at a great distance from the bubble

Symbol	Definition	Unit
c	Sound speed in liquid at infinity	m/s
cp	Heat capacity at constant pressure	J/kg. K
f	Acoustic field frequency	Hz
k	Thermal conductivity	W/m.K
$\dot{m}$	Net rate of evaporation and condensation	$kg/m^2.s$
$\dot{m}_{eva}$	Actual rate of evaporation	$kg/m^2.s$
$\dot{m}_{con}$	Actual rate of condensation	$kg/m^2.s$
P	Pressure	Pa
P_m	Acoustic pressure amplitude	Pa
P^*_v	Saturated liquid pressure	Pa
r	Radial distance from bubble center	m
R	Bubble radius	m
$\dot{R}$	Bubble wall velocity	m/s
R_v	Gas constant of water vapor	J/kg. K
t	Time	s
T	Temperature	K
u	Velocity	m/s
β	Evaporation or condensation accommodation coefficient	
μ	Liquid viscosity	$N.s/m^2$
γ	Ratio of specific heats	
ρ	Density	kg/m^3
σ	Surface tension	N/m
ω	Angular frequency (ω=2πf)	rad/s

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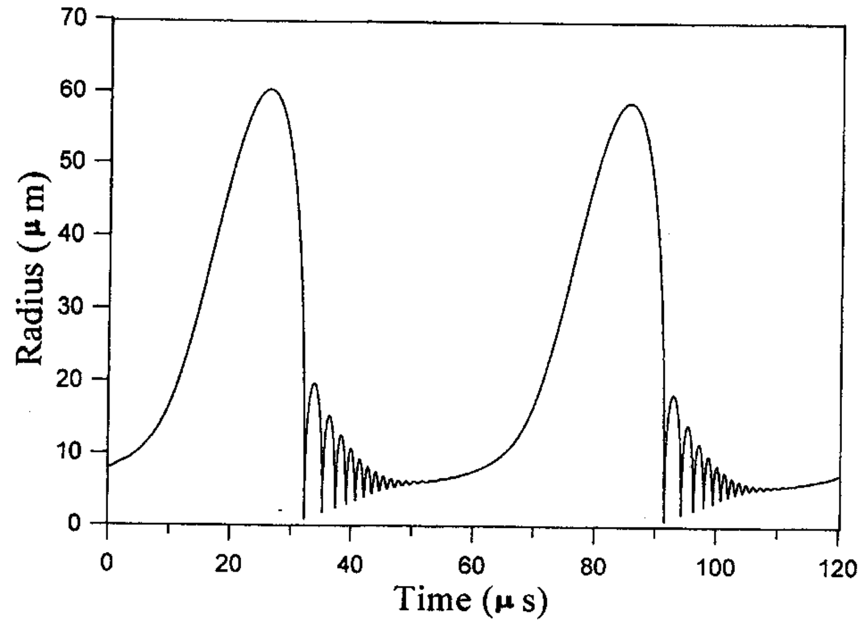

Fig. 1. The bubble radius (R) as a function of time.

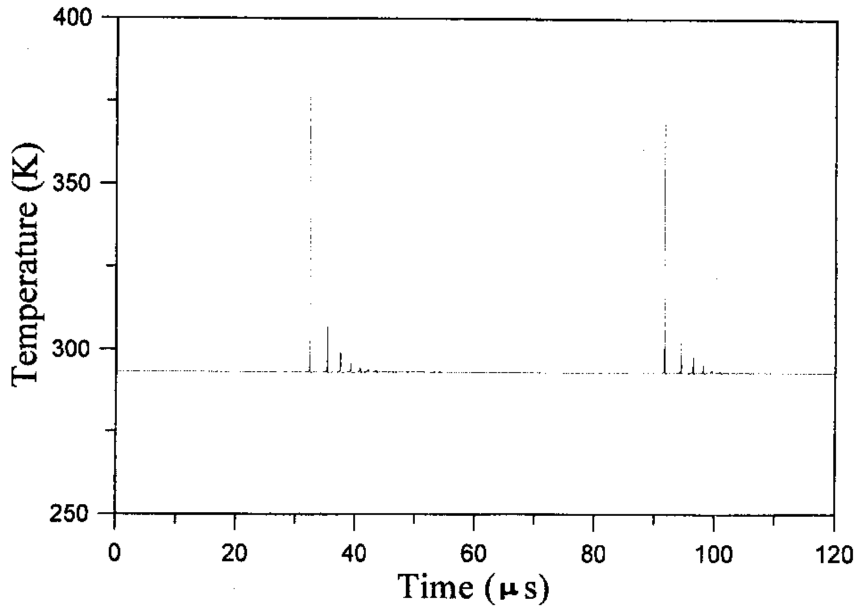

Fig. 2. The liquid temperature at the bubble wall ($T_{L,R}$) as a function of time.

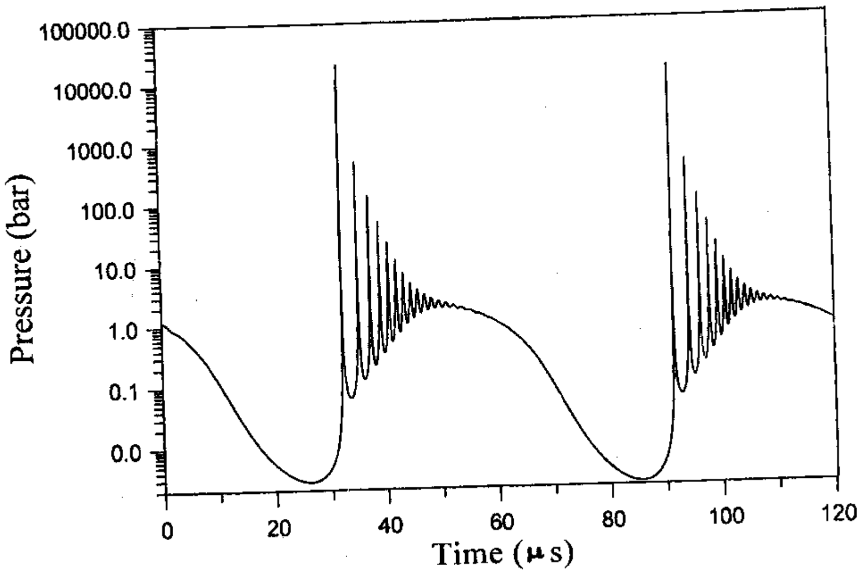

Fig. 3. The pressure inside the bubble (P_V) as a function of time with logarithmic vertical axis.

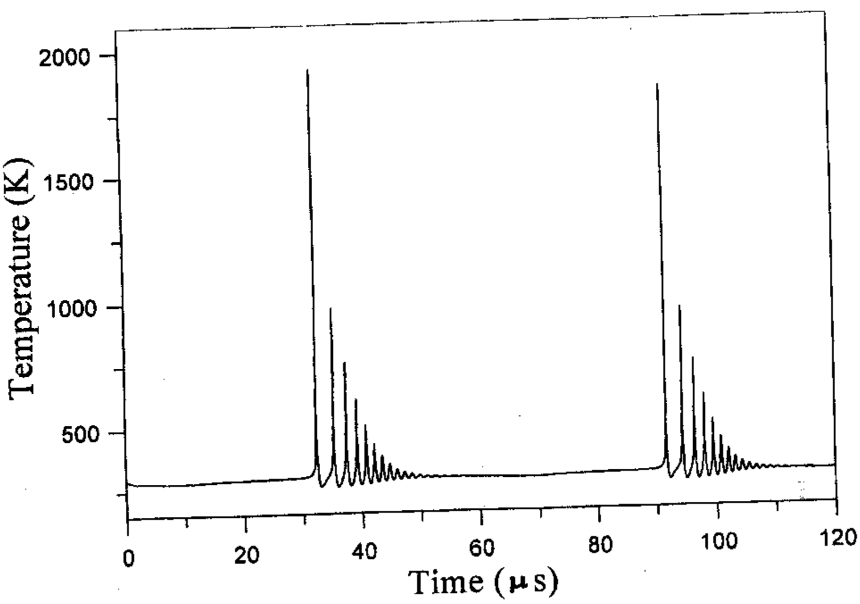

Fig. 4. The temperature at the bubble center as a function of time.

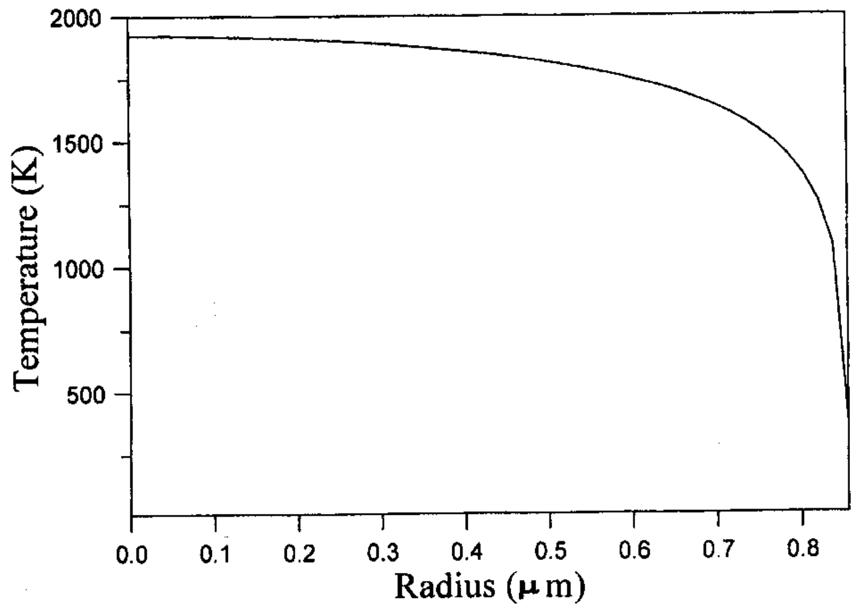

Fig. 5. The temperature distribution inside the bubble at the minimum radius.

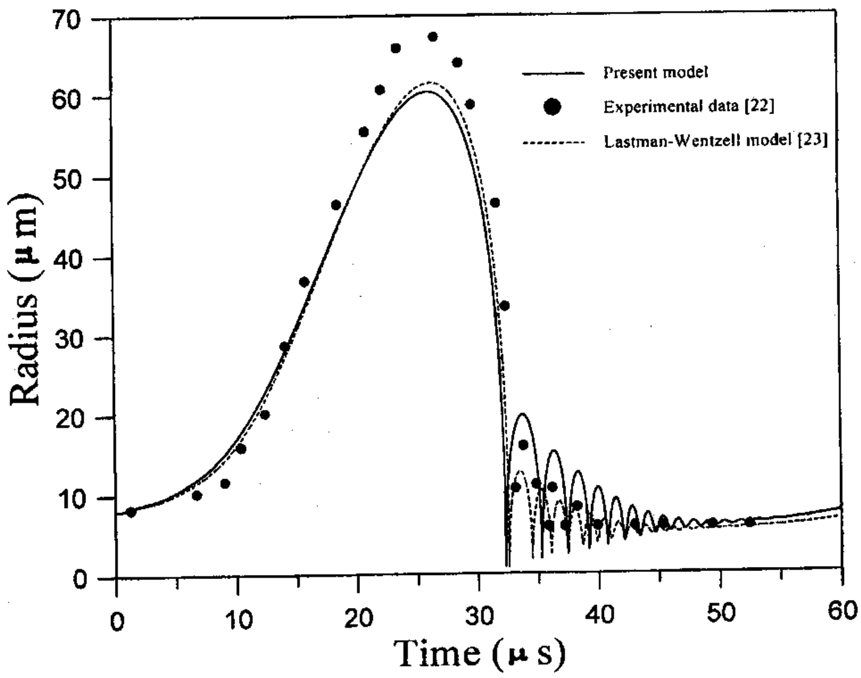

Fig. 6. Comparison between the calculated result and the previous studies of radius-time curve for acoustic cycle.

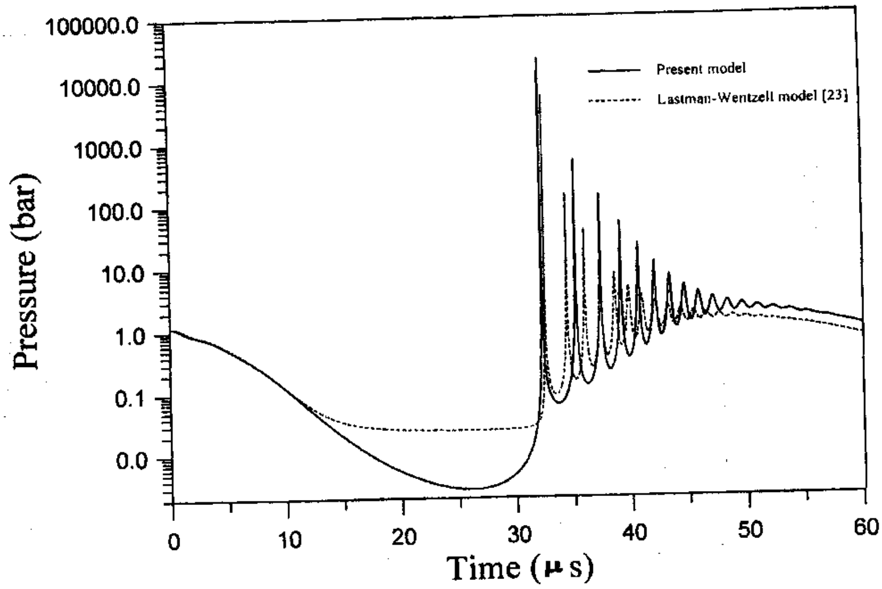

Fig. 7. Comparison between calculated results with previous study of the pressure inside the bubble as a function of time.

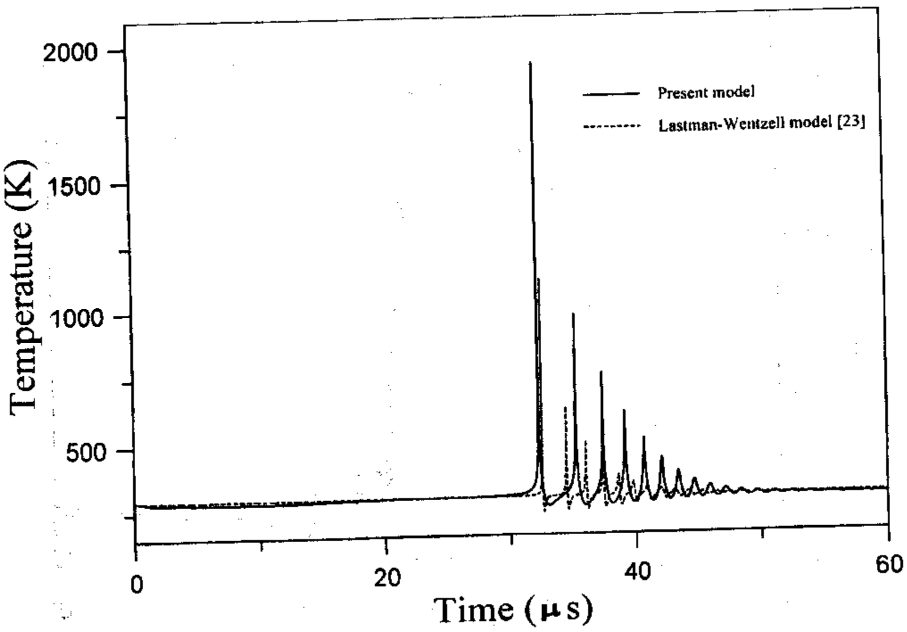

Fig. 8. Comparison between the calculated result with previous study of the temperature at the bubble center as a function of time.